

**NAME:**

Prob #	1	2	3	4	5	6	7	8
Points	6	12	12	15	16	16	14	9



Time: 80 Minutes

**NOTES:**

- a. Credit is only given to the correct numerical values.
- b. All numerical values must be calculated with three digits of accuracy after the decimal point.
- c. Do not write on the back side of the papers.

1. In a ray tracing problem, given the equation of a ray  $\begin{cases} x(t) = -t + 3 \\ y(t) = -4t + 13 \\ z(t) = -2t + 6 \end{cases}$  and a sphere centered at (1,0,2) with radius 5. Find the reflection ray.

Intersection point is (1,5,2).

$$N = (1, 5, 2) - (1, 0, 2) = (0, 5, 0)$$

$$L = (-1, -4, -2)$$

$$R = 2N(N \cdot L) - L$$

2. Given the triangle ABC in a three dimensional right-handed coordinate system, A=(1,1,0), B=(6,1,0), C=(1,5,0)  
Given the intensities at points A=1200 , B=2400 , and C=2800.

Find the intensity at point P(2,3,0) using Gouraud interpolative shading

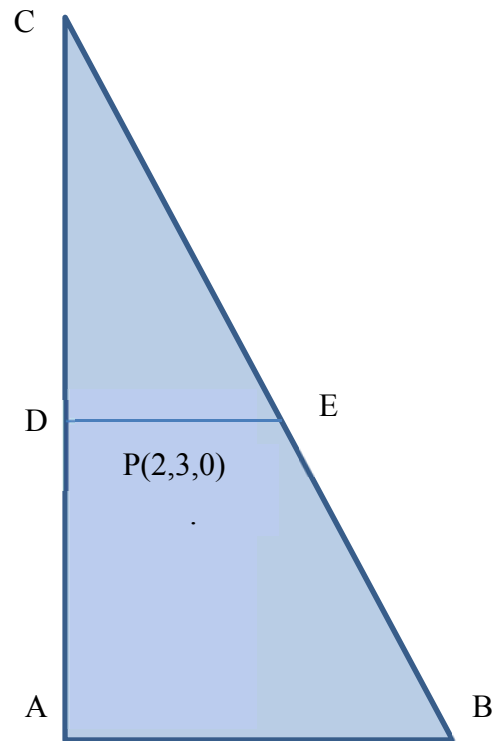
$$v_p = v_1 - (v_1 - v_2) \frac{(P_1 - P)}{(P_1 - P_2)}$$

$$I_D = 2800 - (2800 - 1200) \frac{(5 - 3)}{(5 - 1)} = 2000$$

$$I_E = 2800 - (2800 - 2400) \frac{(5 - 3)}{(5 - 1)} = 2600$$

$$x_E = 6 - (6 - 1) \frac{(5 - 3)}{(5 - 1)} = 3.5$$

$$I_p = 2000 - (2000 - 2600) \frac{(1 - 2)}{(1 - 3.5)} = 2240$$



3. Equation of a parametric surface is given as

$$\begin{aligned} x(u,v) &= 60 u^2 v^2 + 24 uv^2 + 12 u v - 4 v \\ y(u,v) &= 18 u^2 v - 20 u^2 + 12 u v + 10u \\ z(u,v) &= 24 u^2 v^2 + 6 u v - 20 \end{aligned}$$

Find the normalized normal to this surface at point corresponding to  $u=0.5$  and  $v=0.25$

Normal to the surface @  $u=0.5$  and  $v=0.25$  is: \_\_\_\_\_

Normalized normal is: \_\_\_\_\_

$$\begin{cases} \frac{dx}{du} = 120uv^2 + 24v^2 + 12v \\ \frac{dy}{du} = 36uv - 40u + 12v + 10 \\ \frac{dz}{du} = 48uv^2 + 6v \end{cases} \quad \text{and} \quad \begin{cases} \frac{dx}{dv} = 120u^2v + 48uv + 12u - 4 \\ \frac{dy}{dv} = 18u^2 + 12u \\ \frac{dz}{dv} = 48u^2v + 6u \end{cases}$$
  

$$\text{@ } u = 0.5, v = 0.25 \left\{ \begin{array}{l} \frac{dx}{du} = 8.25 \\ \frac{dy}{du} = -2.50 \\ \frac{dz}{du} = 3.00 \end{array} \right. \quad \text{and} \quad \text{@ } u = 0.5, v = 0.25 \left\{ \begin{array}{l} \frac{dx}{dv} = 15.50 \\ \frac{dy}{dv} = 10.50 \\ \frac{dz}{dv} = 6.00 \end{array} \right.$$
  

$$N = [ -46.50 \quad -3.00 \quad 125.375 ] \quad \text{Or} \quad N = [ 46.50 \quad 3.00 \quad -125.375 ]$$
  

$$N = [ -0.348 \quad -0.0224 \quad 0.937 ] \quad \text{Or} \quad N = [ 0.348 \quad 0.0224 \quad -0.937 ]$$

4. Consider a parametric surface  $S(u,v)$ . This surface is quadric in the  $u$  direction and linear in the  $v$  direction. The parametric equations of the two curves at the two boundaries are given:

Parametric equation of the curve corresponding to  $v=0$ :  $C_0(\mathbf{u}) = \mathbf{u}^2 + 2\mathbf{u}$

Parametric equation of the curve corresponding to  $v=1$ :  $C_1(\mathbf{u}) = 3\mathbf{u}^2 + 5\mathbf{u} - 4$

- a. Find the coefficient matrix  $C$  for this surface. (You MUST show the numerical values of the matrix  $C$ . No partial grade will be given for an incomplete solution)

$$S(u, v) = [u^2 \quad u \quad 1] \begin{bmatrix} c_{11} & c_{12} \\ c_{21} & c_{22} \\ c_{31} & c_{32} \end{bmatrix} \begin{bmatrix} v \\ 1 \end{bmatrix}$$

$$S(u, v) = c_{11}u^2v + c_{12}u^2 + c_{21}uv + c_{22}u + c_{31}v + c_{32}$$

$$@v = 0 \quad S(u, 0) = c_{12}u^2 + c_{22}u + c_{32}$$

$$\Rightarrow c_{12} = 1, c_{22} = 2, c_{32} = 0$$

$$@u = 0 \quad S(0, v) = c_{31}v + c_{32}$$

$$\Rightarrow c_{31} = -4$$

$$@v = 1 \quad S(u, 1) = c_{11}u^2 + c_{12}u^2 + c_{21}u + c_{22}u + c_{31} + c_{32}$$

$$\Rightarrow c_{11} = 2, c_{21} = 3$$

- b. Find the equation of the curve corresponding to  $v=0.5$

$$S(u, 0.5) = 2u^2 + 3.5u - 2$$

5. The viewing parameters for a perspective projection are given as:

$$\begin{aligned} \text{VRP(WC)} &= (0,0,0) & \text{VPN(WC)} &= (0, 0,1) \\ \text{VUP(WC)} &= (0,1,0) & \text{PRP (VRC)} &= (2,8,10) \end{aligned}$$

$$\begin{aligned} u_{\min}(\text{VRC}) &= -3 & u_{\max}(\text{VRC}) &= 5 \\ v_{\min}(\text{VRC}) &= -15 & v_{\max}(\text{VRC}) &= 5 \\ n_{\min}(\text{VRC}) &= 13 & n_{\max}(\text{VRC}) &= 15 \end{aligned}$$

Find the sequence of transformations which will transform this viewing volume into a unit cube which is bounded by the planes:  $x=z$  ;  $x=-z$  ;  $y=z$  ;  $y=-z$  ;  $z=z_{\min}$  ;  $z=1$

- Find the **Translation matrix** (Matrix #5)
- Find the **Shear matrix** (Matrix #6)
- Find the **scale matrices** (Matrix #7 and Matrix #8).
- Find the **zmin** after all transformations are done.

**Matrix #2: Rx**

1	0	0	0
0	1	0	0
0	0	1	0
0	0	0	1

**Matrix #4: Rz**

1	0	0	0
0	1	0	0
0	0	1	0
0	0	0	1

**Matrix #6: Shear**

1	0	-0.10	0
0	1	-1.30	0
0	0	1	0
0	0	0	1

**Matrix #8: Scale**


**Matrix #1: Translate**

1	0	0	0
0	1	0	0
0	0	1	0
0	0	0	1

**Matrix #3: Ry**

1	0	0	0
0	1	0	0
0	0	1	0
0	0	0	1

**Matrix #5: Translate**

1	0	0	-2
0	1	0	-8
0	0	1	-10
0	0	0	1

**Matrix #7: Scale**

0.50	0	0	0
0	0.20	0	0
0	0	0.20	0
0	0	0	1

**Zmin = 0.6**

6. Given the triangle ABC in a three dimensional right-handed coordinate system,  $A=(0,0,3)$ ,  $B=(8,0,3)$ ,  $C=(5,6,3)$   
The light source with an intensity of  $I=10000$  is located at  $(7,1,6)$  and the viewer (eye) is located at  $(3,4,7)$  and  $K_a=0$  ;  $K_d=0.5$  ;  $K_s=0.2$  ;  $n=2$

Given point  $P(3,1,3)$  on the triangle ABC:

- a. Find the diffuse intensity at point P

Notes:

Do not use any shading model and ignore  $f_{att}$

$$\vec{AB} = (8,0,3) - (0,0,3) = (8,0,0)$$

$$\vec{AC} = (5,6,3) - (0,0,3) = (5,6,0)$$

$$\vec{N} = \vec{AB} \times \vec{AC} = (0,0,48)$$

$$\hat{N} = (0 \ 0 \ 1)$$

$$\vec{L} = (7,1,6) - (3,1,3) = (4,0,3)$$

$$\hat{L} = (0.8, 0, 0.6)$$

$$I_{Diffuse} = I * k_d * (\hat{N} \cdot \hat{L}) = 10000 * 0.5 * 0.6 = 3000$$

- b. Find the specular intensity at point P from the viewer's point of view

- Do not use any shading model and ignore  $f_{att}$

- 

- $\vec{V} = (3,4,7) - (3,1,3) = (0,3,4)$

- $\hat{V} = (0, 0.6, 0.8)$

- $\hat{R} = 2 * \hat{N} * (\hat{N} \cdot \hat{L}) - \hat{L} = (-0.8 \ 0 \ 0.6)$

- $\hat{R} \cdot \hat{V} = 0.48$

- $I_{Specular} = I * k_s * (\hat{R} \cdot \hat{V})^n = 460.8$

7. Answer each question.

- a) How many triangles are drawn if we use six vertices between glBegin(GL\_TRIANGLE\_FAN) and glEnd()?

Answer:

- b) Given the RGB values of a point, R=0.5, G=0.6, B=0.2, Find the CMYK values of that point:

$$C = 1 - R = 1 - 0.5 = 0.5$$

$$M = 1 - G = 1 - 0.6 = 0.4$$

$$Y = 1 - B = 1 - 0.2 = 0.8$$

$$K = \min(C, M, Y) = 0.4$$

- c) All calls to “glVertex3d” should occur between which two other calls?

- d) The location of a point light influences the ambient intensity.

True

False

- e) For a given point, the diffuse value depends on the location of the observer.

True

False

- f) In the Phong’s illumination model, the higher values of the exponent, n, corresponds to more highlights.

True

False

- g) The origin of the RGB cube (0,0,0) corresponds to Red.

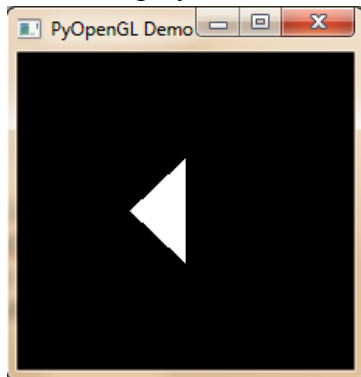
True

False

8. Consider the following OpenGL program:

```
1: from OpenGL.GL import *
2: from OpenGL.GLU import *
3: from OpenGL.GLUT import *
4: def display():
5:     glClear(GL_COLOR_BUFFER_BIT)
6:     glBegin(GL_TRIANGLES)
7:     glColor3f(1,1,1)
8:     glVertex3f(-1,0,0)
9:     glVertex3f(1,0,0)
10:    glVertex3f(0,1,0)
11:    glEnd()
12:    glFlush()
13:    glutSwapBuffers()
14: glutInit(sys.argv)
15: glutInitDisplayMode(GLUT_DOUBLE|GLUT_RGB)
16: glutCreateWindow(b"PyOpenGL Demo")
17: glutDisplayFunc(display)
18: glMatrixMode(GL_PROJECTION)
19: glLoadIdentity()
20: glRotate(90,0,0,1);
21:
22: glFrustum(-1,1,-1,1,1,30)
23: gluLookAt(0,0,3,0,0,0,0,1,0)
24: glMatrixMode(GL_MODELVIEW)
25: glLoadIdentity()
26: glutMainLoop()
```

Which displays the following:





- a. If the code in line 23 is replaced with:

```
gluLookAt(0.0, 0.0, 3.0, 1.0, 0.0, 0.0, 0.0, 1.0, 0.0)
```

What happens to the image on the screen? (all other lines stay the same).

- The size of the image of the object on the screen get larger
- The size of the image of the object on the screen get smaller
- The image of the object on the screen moves down
- The image of the object on the screen moves up
- The image of the object on the screen rotate clockwise
- The image of the object on the screen rotate counter-clockwise
- Nothing changes

- b. If the code in line 22 is replaced with :

```
gluFrustrum(-1, 1, -1, 1, 0.5, 30)
```

What happens to the image on the screen? (all other lines stay the same).

- The size of the image of the object on the screen get larger
- The size of the image of the object on the screen get smaller
- The image of the object on the screen moves to the right
- The image of the object on the screen moves to the left
- The image of the object on the screen rotate clockwise
- The image of the object on the screen rotate counter-clockwise
- Nothing changes

- c. If we add this code to line 21:

```
glRotate(15.0, 0.0, 0.0, 1.0)
```

What happens to the image on the screen? (all other lines stay the same).

- The size of the image of the object on the screen get larger
- The size of the image of the object on the screen get smaller
- The image of the object on the screen moves down
- The image of the object on the screen moves up
- The image of the object on the screen rotate clockwise
- The image of the object on the screen rotate counter-clockwise
- Nothing changes